

Testing the Consistency of Gamma Ray Burst Data-set and Supernovae Union2

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Abstract

In this paper, we test the consistency of Gamma Ray Bursts (GRBs) Data-set and Supernovae Union2 (SNU2) via the so-called *multi-dimensional consistency test* under the assumption that Λ CDM model is a potentially correct cosmological model. We find that the probes are inconsistent with 1.456σ and 85.47% in terms of probability. With this observation, it is concluded that GRBs can be combined with SNU2 to constrain cosmological models.

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I. INTRODUCTION

The cosmic observations of type Ia supernovae (SN Ia) imply that our universe is undergoing an accelerated expansion [1, 2]. Furthermore, this implication was confirmed by the observations from cosmic microwave background radiation [3, 4] and large scale structure [5, 6]. However, understanding the current accelerated expansion of our universe has become one of the most important issues of modern cosmology. In general, from the phenomenological point of view, this late time accelerated expansion of our universe is due to possible modification of gravity theory at large scale or an exotic extra energy component, dubbed dark energy, which has negative pressure.

To reveal the nature of the accelerated expansion or properties of dark energy, one needs more powerful cosmic probes. In the last decade, the data points of SN Ia (the current SNU2) have amounted to the number 557. However, the redshift range of SN Ia is relatively limited. Of course, higher redshift probes are useful to describe the evolution of our universe and to reveal the nature of late time accelerated expansion of our universe and properties of dark energy. The redshift of GRBs can extend to $z \sim 8.1$ or higher which makes it as a complementary cosmic probe to SN Ia. But before using GRBs to constrain cosmological models, the GRBs correlations, which relate cosmological models and intrinsic properties of GRBs, should be calibrated first. In general, the GRBs correlations can be written in a common form of $y = a + bx$ where a and b are the calibrated parameters, x and y are related to the intrinsic properties of GRBs and cosmological models, for the details please see [7]. However, if one calibrates the GRBs correlations via a defined cosmological model, say Λ CDM model with $\Omega_{m0} = 0.27$, the resulting distance moduli of GRBs are not independent of the input cosmology model. As a result, the obtained distance moduli can not be used to constrain any other cosmological models. The so-called circular problem will be committed once the above mentioned results are used to constrain any other cosmological model. Based on this point, the distance modulus derived by Schaefer [7] can not be used to constrain any other cosmological models. So, new methods would be introduced to overcome this problem. Li, *et. al* [8] put the GRBs correlation and its cosmological model constraint together as a whole to fix the calibration parameters and to obtain the best fit values of the cosmological parameters in different cosmological models via Markov Chain Monte Carlo (MCMC) method. The lack of the GRBs calibration makes the GRBs weak to constrain cosmological

models. In fact, the test of the correlations are needed to guarantee the consistency. In an alternative way, cosmography method was considered in [9] by parameterizing the luminosity distance d_L in terms of deceleration parameter q_0 , jerk j_0 and snap s_0 parameters. Clearly, the so-called circular problem is removed. However, this Taylor series method is limited when it is combined with higher redshift data point to constrain cosmological models. Liang *et. al.* [10] used the low redshift SN Ia to calibrate the GRBs correlations and assumed the correlations were respected at high redshifts. Recently, this method was reconsidered by Wei [11, 12]. But, there would be some problems when GRBs are combined with other external data sets to constrain cosmological models. Wang presented a model-independent distance measurement $\bar{r}_p(z_i)$ (Eq. (10) of this paper) from GRBs calibrated internally [13], where z_i are the redshifts of GRBs. The main points of Wang's method are that the resulted distance measurement $\bar{r}_p(z_i)$ is cosmological model in-dependent. The values of correlation parameters a and b are not used directly but the statistical errors of correlation parameters σ_a , σ_b and systematic error σ_{sys} are. This is because the 1σ error bars of a , b and systematic error are almost the same in Λ CDM with different values of Ω_{m0} , though the values of a and b are different. Then, the cosmic constraint from GRBs is obtained in terms of a set of model-independent distance measurements. The merits of this method are listed as follows: (i) the constraint from GRBs is in a cosmological model independent way. So, it can be used to constrain other cosmological models. (ii) It is not calibrated by any other external data sets. It will not suffer any consistent problem when it is combined with other data sets as cosmic constraints. (iii) The cosmological model independent calibration is done first. (iv) Though the absolute calibration of GRBs is not known, the slopes of GRBs correlations can be used as cosmological constraints. Clearly, the drawback is clear that the constrained result is not tighter than the one calibrated by using SN Ia. But, if we have enough data points of GRBs, this problem will be overcome. Because the slopes of GRBs correlations are considered alone, this may make the GRBs not very powerful.

Following the method proposed by Wang [13], Xu obtained $N = 5$ model-independent distances data sets and their covariance matrix by using 109 GRBs via Amati's $E_{p,i} - E_{iso}$ correlation [23]. These five model-independent distances data points have been used to constrain cosmological model [24]. However, the consistency of the obtained five data sets via Amati's correlation with other cosmic probes must be checked to guarantee the reliability of GRBs. With this motivation, we will test the consistency or inconsistency of SNU2 with

these five data sets derived from GRBs via the so-called *multi-dimensional consistency test* which will be reviewed briefly in section II C, for the details please see [25].

This paper is structured as follows. In section II, the SNU2, the five data sets derived from GRBs and the method to constrain dark energy model are presented. Also, the multi-dimensional consistency test is reviewed briefly. Section III is the concluding remark.

II. DATA-SETS AND METHOD

A. Type Ia Supernovae

Recently, SCP (Supernova Cosmology Project) collaboration released their Union2 dataset which consists of 557 SN Ia [20]. The distance modulus $\mu(z)$ is defined as

$$\mu_{th}(z) = 5 \log_{10}[\bar{d}_L(z)] + \mu_0, \quad (1)$$

where $\bar{d}_L(z)$ is the Hubble-free luminosity distance $H_0 d_L(z)/c = H_0 d_A(z)(1+z)^2/c$, with H_0 the Hubble constant, and $\mu_0 \equiv 42.38 - 5 \log_{10} h$ through the re-normalized quantity h as $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$. Where $d_L(z)$ is defined as

$$d_L(z) = (1+z)r(z), \quad r(z) = \frac{c}{H_0 \sqrt{|\Omega_k|}} \text{sinn} \left[\sqrt{|\Omega_k|} \int_0^z \frac{dz'}{E(z')} \right] \quad (2)$$

where $E^2(z) = H^2(z)/H_0^2$. Additionally, the observed distance moduli $\mu_{obs}(z_i)$ of SN Ia at z_i are

$$\mu_{obs}(z_i) = m_{obs}(z_i) - M, \quad (3)$$

where M is their absolute magnitudes.

For the SN Ia dataset, the best fit values of the parameters p_s can be determined by a likelihood analysis, based on the calculation of

$$\begin{aligned} \chi^2(p_s, M') &\equiv \sum_{SN} \frac{\{\mu_{obs}(z_i) - \mu_{th}(p_s, z_i)\}^2}{\sigma_i^2} \\ &= \sum_{SN} \frac{\{5 \log_{10}[\bar{d}_L(p_s, z_i)] - m_{obs}(z_i) + M'\}^2}{\sigma_i^2}, \end{aligned} \quad (4)$$

where $p_s = \{\Omega_{m0}\}$ denotes the model parameter and $M' \equiv \mu_0 + M$ is a nuisance parameter which includes the absolute magnitude and the parameter h . The nuisance parameter M'

can be marginalized over analytically [21] as

$$\bar{\chi}^2(p_s) = -2 \ln \int_{-\infty}^{+\infty} \exp \left[-\frac{1}{2} \chi^2(p_s, M') \right] dM',$$

resulting to

$$\bar{\chi}^2 = A - \frac{B^2}{C} + \ln \left(\frac{C}{2\pi} \right), \quad (5)$$

with

$$\begin{aligned} A &= \sum_{i,j}^{SN} \left\{ 5 \log_{10}[\bar{d}_L(p_s, z_i)] - m_{obs}(z_i) \right\} \cdot \text{Cov}_{ij}^{-1} \cdot \left\{ 5 \log_{10}[\bar{d}_L(p_s, z_j)] - m_{obs}(z_j) \right\}, \\ B &= \sum_i^{SN} \text{Cov}_{ij}^{-1} \cdot \left\{ 5 \log_{10}[\bar{d}_L(p_s, z_j)] - m_{obs}(z_j) \right\}, \\ C &= \sum_i^{SN} \text{Cov}_{ii}^{-1}, \end{aligned} \quad (6)$$

where Cov_{ij}^{-1} is the inverse of covariance matrix with or without systematic errors. One can find the details in Ref. [20] and the web site ¹ where the covariance matrix with or without systematic errors are included. Relation (4) has a minimum at the nuisance parameter value $M' = B/C$, which contains information of the values of h and M . Therefore, one can extract the values of h and M provided the knowledge of one of them. Finally, the expression

$$\chi_{SN}^2(p_s, B/C) = A - (B^2/C), \quad (7)$$

which coincides to Eq. (5) apart from a constant, is often used in the likelihood analysis [21, 22]. Thus in this case the results will not be affected by a flat M' distribution. It is worth noting that the results will be different with or without the systematic errors. In this work, all results are obtained with systematic errors.

B. Gamma Ray Bursts

Following [7], we consider the well-known Amati's $E_{p,i} - E_{iso}$ correlation [14, 16–18] in GRBs, where $E_{p,i} = E_{p,obs}(1+z)$ is the cosmological rest-frame spectral peak energy, and E_{iso} is the isotropic energy

$$E_{iso} = 4\pi d_L^2 S_{bolo}/(1+z) \quad (8)$$

¹ <http://supernova.lbl.gov/Union/>

in which d_L and S_{bolo} are the luminosity distance and the bolometric fluence of the GRBs respectively. Following [7], we rewrite the Amati's relation as

$$\log \frac{E_{iso}}{\text{erg}} = a + b \log \frac{E_{p,i}}{300\text{keV}}. \quad (9)$$

In [13], Wang defined a set of model-independent distance measurements $\{\bar{r}_p(z_i)\}$:

$$\bar{r}_p(z_i) \equiv \frac{r_p(z)}{r_p(z_0)}, \quad r_p(z) \equiv \frac{(1+z)^{1/2}}{z} \frac{H_0}{c} r(z), \quad (10)$$

where $r(z) = d_L(z)/(1+z)$ is the comoving distance at redshift z , and z_0 is the lowest GRBs redshift. Then, the cosmological model can be constrained by GRBs via

$$\chi_{GRBs}^2(p_s) = [\Delta \bar{r}_p(z_i)] \cdot (Cov_{GRB}^{-1})_{ij} \cdot [\Delta \bar{r}_p(z_i)], \quad (11)$$

$$\Delta \bar{r}_p(z_i) = \bar{r}_p^{data}(z_i) - \bar{r}_p(z_i), \quad (12)$$

where $\bar{r}_p(z_i)$ is defined by Eq. (10) and $(Cov_{GRB}^{-1})_{ij}, i, j = 1 \dots N$ is the covariance matrix. In this way, the constraints from observational GRBs data are projected into the relative few quantities $\bar{r}_p(z_i), i = 1 \dots N$.

Following the method proposed by Wang [13], Xu obtained $N = 5$ model-independent distances data sets and their covariance matrix by dividing 109 GRBs into five bins via Amati's $E_{p,i} - E_{iso}$ correlation [23]. The resulting model-independent distances and covariance matrix from 109 GRBs are shown below in Tab. I and Eq. (14). The $\{\bar{r}_p(z_i)\} (i = 1, \dots, 5)$

	z	$\bar{r}_p^{data}(z)$	$\sigma(\bar{r}_p(z))^+$	$\sigma(\bar{r}_p(z))^-$
0	0.0331	1.0000	—	—
1	1.0000	0.9320	0.1711	0.1720
2	2.0700	0.9180	0.1720	0.1718
3	3.0000	0.7795	0.1630	0.1629
4	4.0480	0.7652	0.1936	0.1939
5	8.1000	1.1475	0.4297	0.4389

TABLE I. Distances measured from 109 GRBs via Amati's correlation with 1σ upper and lower uncertainties [23]. $z_0 = 0.0331$ as lowest redshift was adopted.

correlation matrix is given by

$$(\overline{Cov}_{GRB}) = \begin{pmatrix} 1.0000 & 0.7780 & 0.8095 & 0.6777 & 0.4661 \\ 0.7780 & 1.0000 & 0.7260 & 0.6712 & 0.3880 \\ 0.8095 & 0.7260 & 1.0000 & 0.6046 & 0.5032 \\ 0.6777 & 0.6712 & 0.6046 & 1.0000 & 0.1557 \\ 0.4661 & 0.3880 & 0.5032 & 0.1557 & 1.0000 \end{pmatrix}, \quad (13)$$

and the covariance matrix is given by

$$(Cov_{GRB})_{ij} = \sigma(\bar{r}_p(z_i))\sigma(\bar{r}_p(z_j))(\overline{Cov}_{GRB})_{ij}, \quad (14)$$

where

$$\sigma(\bar{r}_p(z_i)) = \sigma(\bar{r}_p(z_i))^+, \quad \text{if } \bar{r}_p(z) \geq \bar{r}_p(z)^{data}; \quad (15)$$

$$\sigma(\bar{r}_p(z_i)) = \sigma(\bar{r}_p(z_i))^- , \quad \text{if } \bar{r}_p(z) < \bar{r}_p(z)^{data}, \quad (16)$$

the $\sigma(\bar{r}_p(z_i))^+$ and $\sigma(\bar{r}_p(z_i))^-$ are the 1σ errors listed in Tab. I.

C. Method: Multi-dimensional Consistency Test

In Ref. [25], the multi-dimensional consistency test of probes was considered. If we consider M parameters and N probes, the method to test the consistency is to minimize the $\chi^2(\lambda_\alpha)$ with respect to λ_α ,

$$\chi^2(\lambda_\alpha) = \sum_{i=1}^M \sum_{\alpha,\beta=1}^N (\lambda_\alpha - \lambda_\alpha^{(i)}) [C^{(i)}]_{\alpha\beta}^{-1} (\lambda_\beta - \lambda_\beta^{(i)}), \quad (17)$$

where $\lambda_\alpha^{(i)}$ is the best fit value returned from the i th probe with covariance matrix $C_{\alpha\beta}^{(i)}$, and λ_α is a random point in cosmological space. In this case, the value of λ_α at the minimum χ^2 is the best fit value. The goodness of fit is quantified by the value of χ^2 in the standard way, i.e., by checking the expectation value $\langle \chi_{min}^2 \rangle = \nu$ where ν is the degrees of freedom. For example, in our case, we consider the Λ CDM model with Ω_{m0} as a free model parameter ($M = 1$) and two probes ($N=2$): SN and GRBs. So, the degrees of freedom ($\nu = N - M$) are $2 - 1 = 1$. The expectation value of χ_{min}^2 would be 1. However, the value $\langle \chi_{min}^2 \rangle = \nu + B$ will be returned, where $B > 0$ denotes the possible deviation from ν . Then, the consistency can be concluded by the value of d_σ via the formula

$$1 - \Gamma(\nu/2, B/2)/\Gamma(\nu/2) = \text{Erf}(d_\sigma/\sqrt{2}). \quad (18)$$

The larger value of d_σ denotes better inconsistency between the probes. For example, a difference $B = 9$ tells us the two probes are inconsistent with 99.7% (3σ) in Λ CDM model. For convenience, we show the relation between B and probability in Fig. 1 where $\nu = 1$ is adopted.

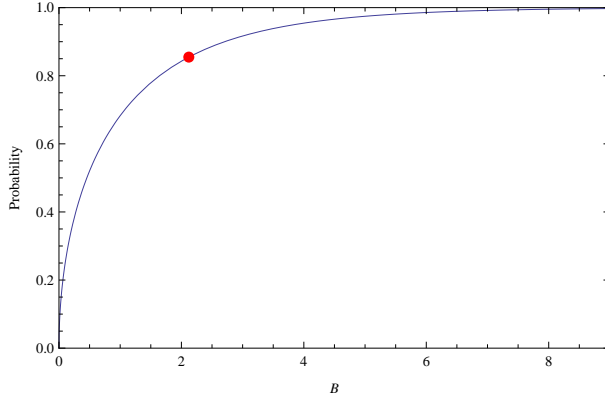


FIG. 1. The relation between B and probability where $\nu = 1$ is adopted, where the red point denotes the probability 85.47% at $B = 2.121$.

As described above, we firstly find the corresponding minimum χ^2_{min} values with SN and GRBs via Markov Chain Monte Carlo (MCMC) method. Our code is based on the publicly available **CosmoMC** package [26]. The results are shown in Tab. II. Via the formula (17), we find the minimum value of $\chi^2(\Omega_{m0})$ is 3.121 with the best fit value of $\Omega_{m0} = 0.287$, where the covariance matrix $C_{\alpha\beta}^{(i)}$ is given

$$C_{\alpha\beta}^{(i)} = \sigma_\alpha^i(\lambda^i)\sigma_\beta^i(\lambda^i)\delta_{\alpha\beta}, \quad (19)$$

here

$$\sigma_\alpha^i(\lambda^i) = \sigma_\alpha^i(\lambda^i)^+, \quad \text{if } \lambda_\alpha \geq \lambda_\alpha^i; \quad (20)$$

$$\sigma_\alpha^i(\lambda^i) = \sigma_\alpha^i(\lambda^i)^-, \quad \text{if } \lambda_\alpha < \lambda_\alpha^i, \quad (21)$$

and the correlation between SN and GRBs is zero. However, the expected value of $\chi^2(\Omega_{m0})$ should be $\nu = 1$. Then the returned deviation B is 2.121. From Eq. (18), one finds that the probes are inconsistent with $d_\sigma = 1.456\sigma$ and 85.47% in terms of probability.

Datasets	Parameters	$\chi^2_{min}/\text{d.o.f}$	Ω_{m0}
SNU2	1	530.722/556	$0.274^{+0.0386}_{-0.0358}$
GRB	1	3.0354/4	$0.620^{+0.313}_{-0.192}$

TABLE II. The results of χ^2_{min} , Ω_{m0} with 1σ regions are listed, where SN systematic errors are included. d.o.f denotes the degrees of freedom.

III. CONCLUSION

The consistency of SN Ia and GRBs is checked by the so-called *multi-dimensional consistency test* under the assumption that Λ CDM model is a correct cosmological model. We find that the inconsistency of SNU2 and GRBs is about 1.456σ and 85.47% in terms of probability. So, we can conclude that the five GRBs data points are consistent with SNU2 at the level above 1.456σ . These five GRBs data sets can be combined with SNU2 to constrain other cosmological models.

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